Biased Random-Key Genetic Algorithms

Algoritmos Genéticos de Chaves Aleatórias Viciadas

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Este material foi desenvolvido baseando-se em
http://mauricio.resende.info/talks/2012-09-CLAIO2012-brkga-tutorial-both-days.pdf

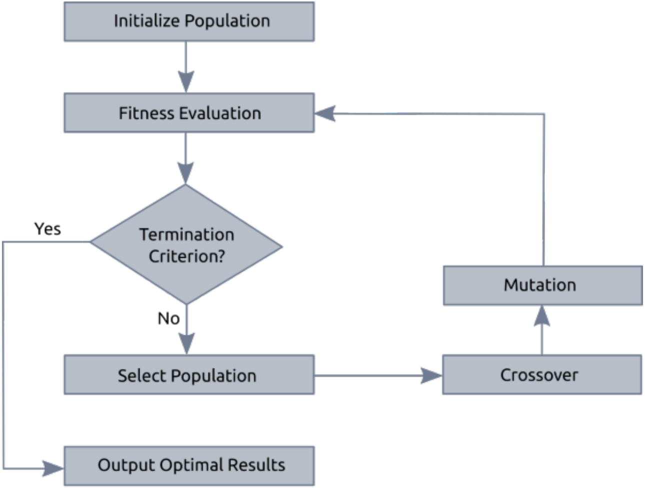
Summary

1. Genetic algorithms (GAs)
2. Random-key genetic algorithms (RKGAs)
3. Biased random-key genetic algorithm (BRKGAs)

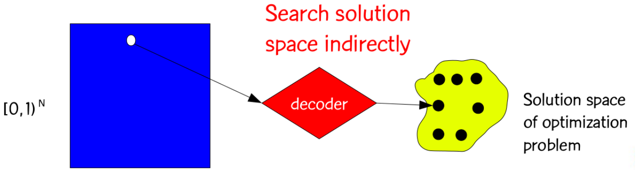
- Genetic algorithms (GAs) are metaheuristics inspired by the process of natural selection
- GAs evolve population of individuals (solutions) applying Darwin's principle of survival of the fittest
- A GA maintains a population of candidate individuals (solutions) for the problem at hand, and makes it evolve by iteratively applying a set of **stochastic operators** (selection, recombination and mutation)
    - selection: replicates the most successful individuals found in a population at a rate proportional to their relative quality
    - recombination (crossover): decomposes two or more distinct individuals and then randomly mixes their parts to form novel solutions
    - mutation: randomly perturbs a candidate individual

- Encoding principles (gene, chromosome)
- Initialization procedure (creation)
- Selection of parents (reproduction)
- Genetic operators (mutation, recombination)
- Evaluation function (environment)
- Termination condition

Genetic algorithms
Evolutionary cycle



- Introduced by Bean (1994)[1] for sequencing problems
- A random-key is a real random number in the continuous interval [0,1)
- Individuals (solutions) of optimization problems can be encoded by random-keys
- Individuals are strings of real-valued numbers (random-keys)
- A decoder is a deterministic algorithm that takes a vector of random-keys as input and outputs a solution of the optimization problem



[1] Bean, J. C. (1994). Random-key genetic algorithms for sequencing and optimization. ORSA journal on computing, 6(2), 154-160.

Random-key genetic algorithms

Encoding/decoding principles

- Bean (1994)[1] proposed decoders based on sorting the random-key vector to produce a sequence

- Encoding:

$$s = \langle \underset{1}{0.25}, \underset{2}{0.19}, \underset{3}{0.67}, \underset{4}{0.05}, \underset{5}{0.89} \rangle$$

- Decode by sorting vector of random-keys:

$$s' = \langle \underset{4}{0.05}, \underset{2}{0.19}, \underset{1}{0.25}, \underset{3}{0.67}, \underset{5}{0.89} \rangle$$

- Therefore, the vector of random-keys:

$$s = \langle 0.25, 0.19, 0.67, 0.05, 0.89 \rangle$$

encodes the sequence: $\langle 4, 2, 1, 3, 5 \rangle$

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    $$s = \langle \underset{1}{0.099}, \underset{2}{0.216}, \underset{3}{0.802}, \underset{4}{0.368}, \underset{5}{0.658} \rangle$$

    decode by sorting vector of random-keys:

    $$s' = \langle \underset{1}{0.099}, \underset{2}{0.216}, \underset{4}{0.368}, \underset{5}{0.658}, \underset{3}{0.802} \rangle$$

    encodes the subset: $\{1, 2, 4\}$

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    encodes the subset: $\{1, 2, 4\}$
  - subset selection (select k of 5 elements, where $0 \leq k \leq 5$)

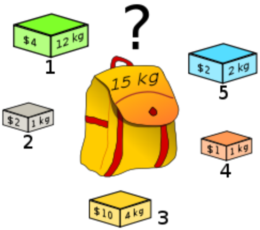
    encoding:

    $$s = \langle \underset{1}{0.82}, \underset{2}{0.12}, \underset{3}{0.54}, \underset{4}{0.89}, \underset{5}{0.26} \rangle$$

    decoding: if $s_i \geq 0.5$ then select i

    encodes the subset: $\{1, 3, 4\}$

- For some cases of complex decoders is necessary to adjust the chromosome in order to produce a feasible solution
- Knapsack problem:



encoding:

$$s = \langle \underset{1}{0.82}, \underset{2}{0.12}, \underset{3}{0.54}, \underset{4}{0.89}, \underset{5}{0.26} \rangle$$

decoding: if $s_i \geq 0.5$ then get item i

encodes the subset items: $\{1, 3, 4\}$

$\sum_{i=1,3,4} w_i \geq W \ \ (12 + 4 + 1 \geq 15)$

so, a **deterministic** strategy must be applied to make the solution viable

Random-key genetic algorithms

Initialize population

- Initial population is made up of P random-key vectors, each with N keys, each having a value generated uniformly at random in the interval [0,1).

$$s_1 = \langle \text{KEY}_1^1, \text{KEY}_2^1, \cdots, \text{KEY}_N^1 \rangle$$
$$s_2 = \langle \text{KEY}_1^2, \text{KEY}_2^2, \cdots, \text{KEY}_N^2 \rangle$$
$$\cdots$$
$$s_P = \langle \text{KEY}_1^P, \text{KEY}_2^P, \cdots, \text{KEY}_N^P \rangle$$

$$\text{KEY}_j^i \leftarrow \text{rand}\,[0,1) \qquad \forall i = 1..P, \forall j = 1..N$$

Random-key genetic algorithms

Selection of parents and recombination

- Two parents a and b are randomly selected from the entire P population
- Mating is done using parameterized uniform crossover (Spears & DeJong , 1990)[2] for sequencing problems.

$$a = \langle 0.25, 0.19, 0.67, 0.05, 0.89 \rangle$$
$$b = \langle 0.63, 0.90, 0.76, 0.93, 0.08 \rangle$$

- For each gene, flip a biased coin to choose which parent passes the allele (key, or value of gene) to the child.

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If every random-key array corresponds to a feasible solution: Mating always produces feasible offspring.

[2] Spears, W. M., & De Jong, K. D. (1995). On the virtues of parameterized uniform crossover. NAVAL RESEARCH LAB WASHINGTON DC.

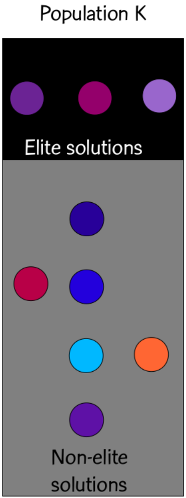
- **No mutation**: mutants are used instead (they play same role as mutation in GAs ... help escape local optima)

- A mutant is a new individual generated uniformly at random in the interval [0,1)

$$m = \langle \mathrm{KEY}_1^m, \mathrm{KEY}_2^m, \cdots, \mathrm{KEY}_N^m \rangle$$

$$\mathrm{KEY}_j^m \leftarrow \mathrm{rand}\,[0, 1) \qquad \forall j = 1..N$$

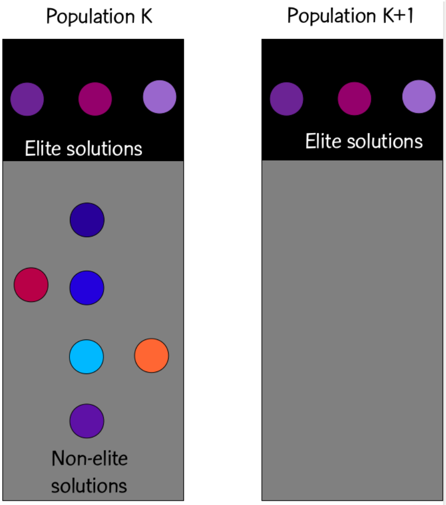
- At the k-th generation, compute the cost of each solution and partition the individuals into two sets:
  - elite individuals (P_e)
  - non-elite individuals ($\overline{P}_e = P \setminus P_e$)
- Elite set should be smaller of the two sets and contain best individuals, i.e., $|P_e| < |P|/2$



Random-key genetic algorithms

Next generation

- Copy elite individuals P_e from population k to population $k+1$



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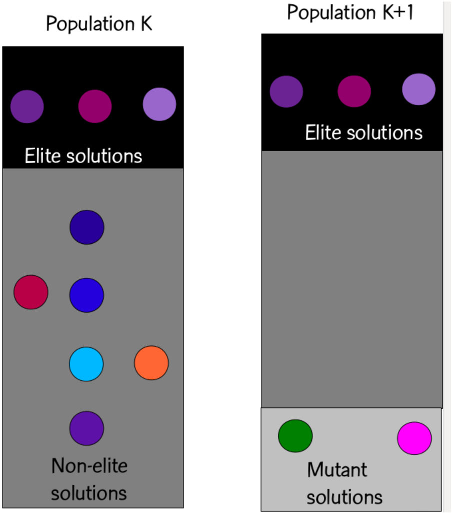
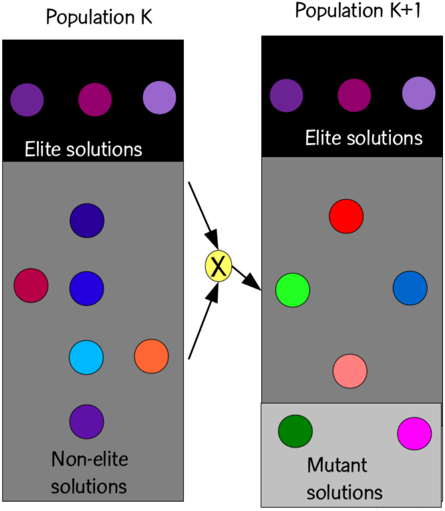
Next generation

- Copy elite individuals P_e from population k to population $k+1$
- Add $|P_m|$ random individuals (mutants) to population $k+1$



- Copy elite individuals P_e from population k to population $k+1$
- Add $|P_m|$ random individuals (mutants) to population $k+1$
- While $k+1$-th population $< P$ **do**
  - Use any two individuals in population k to produce child in population $k+1$. Mates are chosen at random.



- A biased random-key genetic algorithm (BRKGA) is a random-key genetic algorithm (RKGA).
- BRKGA and RKGA differ in how mates are chosen for crossover and how parameterized uniform crossover is applied.

Biased random-key genetic algorithm

Selection of parents and recombination

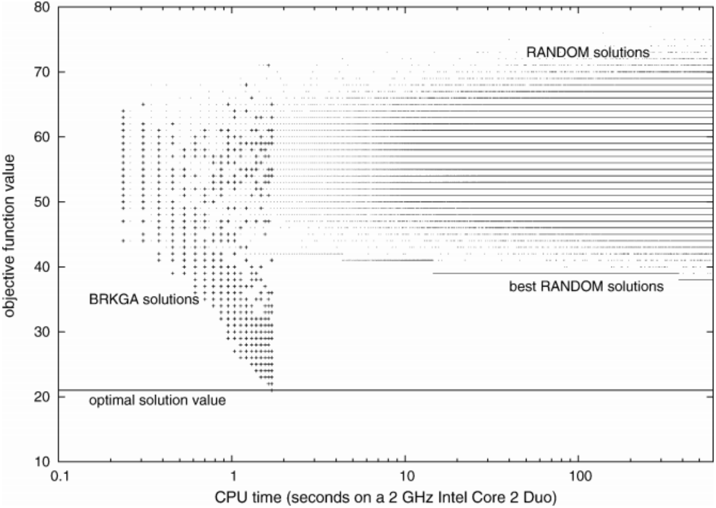
- A parent a is randomly selected from the elite population P_e
- A parent b is randomly selected from the ~~elite~~ non-elite population $\overline{P}_e$ **or** is randomly selected from entire population P
- For $i = 1, ..., n$, the i-th allele c_i of the offspring c takes on the value of the i-th allele a_i of the elite parent a with probability ρ_e and the value of the i-th allele b_i of the non-elite parent b with probability $1 - \rho_e$

Parent *a*	0.32	0.77	0.53	0.85
Parent *b*	0.26	0.15	0.91	0.44
Random number	0.58	0.89	0.68	0.25
$\rho_e = 0.7$	$<$	$>$	$<$	$<$
Offspring *c*	0.32	0.15	0.53	0.85

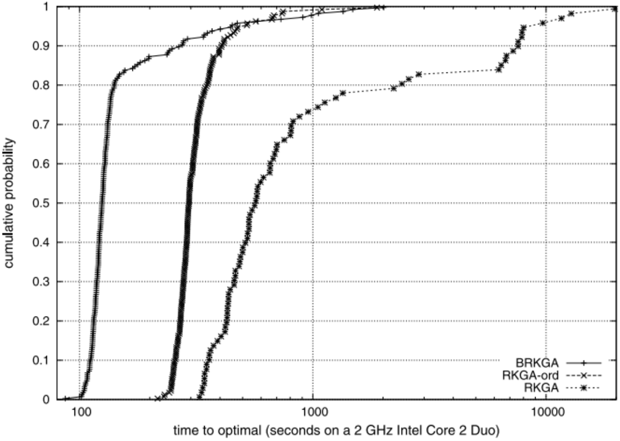
- In this way, the offspring is more likely to inherit characteristics of the elite parent than those of the non-elite parent.

Biased random-key genetic algorithm

Random solutions vs BRKGA solutions



Comparing a BRKGA with a random multistart heuristic on an instance of a covering by pairs problem

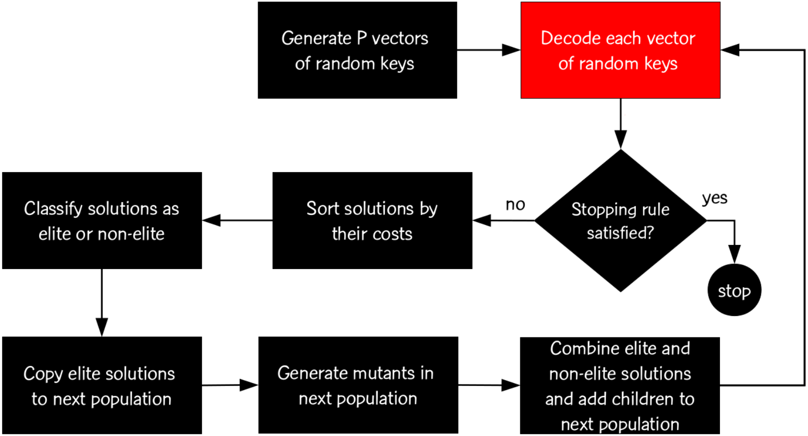


Time to target plots compare running times needed to find the optimal solution of a 220 element covering by pairs problem with a BRKGA and two variants of a RKGA

RKGA-ord is similar to a RKGA except that the offspring inherit the allele of the better fit of the two parents with probability ρ_e

Biased random-key genetic algorithm

Framework



Biased random-key genetic algorithm

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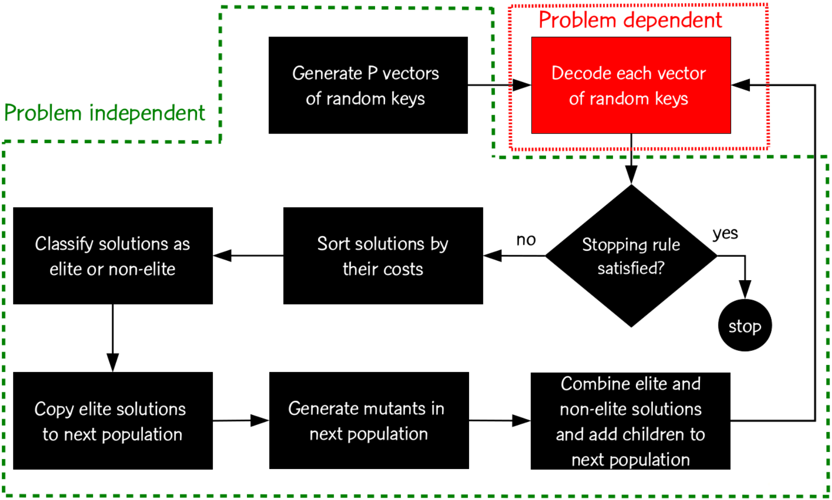
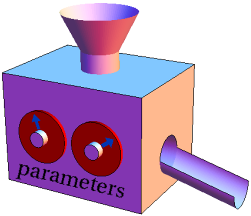


Biased random-key genetic algorithm

Parameters



Biased random-key genetic algorithm

Parameters

Parameter	Description	Recommended value
P	size of population	$P = aN$, where $1 \leq a \in \mathbb{R}$ is a constant and N is the length of the chromosome
P_e	size of elite population	$0.10P \leq P_e \leq 0.25P$
P_m	size of mutant population	$0.10P \leq P_m \leq 0.30P$
ρ_e	elite allele inheritance probability	$0.5 < \rho_e \leq 0.8$
$STOP$	stopping criterion	e.g. time, # generations, solution quality, # generations without improvement

- Efficient and easy-to-use object oriented application programming interface (API) for the algorithmic framework of BRKGA
- Download: `http://mauricio.resende.info/src/brkgaAPI/`

References

- Resende, M. G. (2012). Biased random-key genetic algorithms: A tutorial
  http://mauricio.resende.info/talks/2012-09-CLAIO2012-brkga-tutorial-both-days.pdf
- Gonçalves, J. F., & Resende, M. G. (2011). Biased random-key genetic algorithms for combinatorial optimization. Journal of Heuristics, 17(5), pp. 487-525.